DFPD/99/TH/45
February 1, 2008

Macroscopic Forces driven by Resonant Neutrino Conversion

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Abstract

We show that neutrino oscillations in matter are always accompanied by collective forces on the medium. This effect may produce interesting consequences for the background and the neutrino oscillations themselves. The force is maximal in the case of resonant neutrino conversion in the adiabatic regime. We study here the forces driven by $\nu_e - \nu_{\mu,\tau}$ and $\nu_e - \nu_s$ MSW conversion and shortly discuss their possible relevance for the dynamics of a type II supernova.

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It is well known that the neutrino propagation in matter can be drastically altered if neutrinos carry non-standard properties like non-vanishing mass, mixing, magnetic moment, and new interactions with the matter constituents and the classical fields. The MSW neutrino resonant conversion [1] and the resonant spin-flavour precession (RSFP)[2] are examples of phenomena that can take place due to the interplay of the coherent interaction of neutrinos with the matter and the anomalous neutrino properties. Usually, the study of such phenomena is focused on the fate of the neutrinos disregarding any possible implications for the medium itself. However, such an attitude could be not always justified especially in those cases where the neutrino flux is very intense, like during a type II supernova (SN) explosion [3].

Recent studies [4, 5] showed that the coherent interaction of neutrinos with the matter constituents can gives rise to macroscopic forces even in the case in which neutrinos are massless and they have only standard interactions. Such forces are usually called *ponderomotive forces* in analogy to the forces that arise in an electron gas in the presence of a non-uniform radiation field [6, 7]. A similar phenomenon takes place for the electrons, and other weakly interacting matter constituents, in the presence of nonuniform flux of neutrinos. According to the approach of Hardy and Melrose [4], the background of neutrinos gives rise to a space-dependent self-energy, hence to a space-dependent mass correction, of the matter particles. By interpreting the statistical average of the mass correction as an interaction energy density $\mathcal{U}$, Hardy and Melrose deduced the existence of a ponderomotive force per unit volume $\mathcal{F} = -\nabla\mathcal{U}$ which is proportional to the Fermi constant G_F. The reader should not confuse the ponderomotive force with the more conventional force produced on the matter constituents by the incoherent elastic scattering off the neutrinos, which is proportional to G_F^2. In the past years some work has been done concerning collective forces produced by coherent neutrino scattering in relation to possible detection of relic neutrinos [8]. On the basis of the Born approximation and the assumption of spatially homogeneous neutrino flux, it was proved that to first order in G_F the effect is vanishing [8]. We note, however, that these arguments do not apply to objects of astrophysical sizes where the neutrino flux is not homogeneous.

The relevance of neutrino induced ponderomotive forces for the physics of type II supernovae and other extreme astrophysical objects was investigated by several authors [4, 5, 9, 10]. In some cases, non-negligible effects were predicted.

The aim of this Letter is to show that neutrino oscillations in matter are always accompanied by collective forces on the background medium. Furthermore, we will show that in some phenomenological interesting cases, these forces are quite larger than the ponderomotive forces generated in the absence of neutrino oscillations. After deriving the general expression of the force produced by neutrino oscillations we shall focus on the case of MSW neutrino conversion. As an application, we shall briefly discuss the possible relevance of our results for the dynamics of a type II SN.

Before entering into the details of our derivation, we have to mention that in the last few years some debate was going on concerning the correct expression of the ponderomotive force in the absence of neutrino oscillations [11]. Since our results might be only marginally affected by the outcome of this controversy, we do not enter here into

such discussion. We observe only that our general treatment lead to results that are consistent with those of Hardy and Melrose.

Similarly to what done by other authors, we determine the ponderomotive force per unit volume by taking the gradient of the interaction energy density $\mathcal{U}$. The way we compute $\mathcal{U}$ is however more straightforward than that followed in Ref.[4] and it allows us to apply the formalism which is commonly used to treat neutrino oscillations in matter. For the sake of clarity we assume here that the relevant components of the medium are well described by some equilibrium distribution function. We also assume that at the position $\mathbf{x}$, where the force is computed, neutrinos are already thermally decoupled from the background. According to the Liouville theorem neutrinos can still be described by a Fermi-Dirac distribution function $f_{\nu_a}(\mathbf{x};\mathbf{k})$ (where the variable $\mathbf{x}$ enters as a parameter) with an effective temperature which will depend on the distance from the decoupling position. The interaction energy density of the neutrinos with the $i-th$ medium component is computed by taking the thermal average of the interaction Hamiltonian over the fermion distributions. We have

$$\mathcal{U}_i(\mathbf{x}) = \sum_{a,b=e,\mu,\tau} \int \frac{d^3k}{(2\pi)^3} f_{\nu_a}(\mathbf{x};\mathbf{k}) P_{\nu_a\nu_b}(\mathbf{x},\mathbf{k}) V_{\nu_b i}(\mathbf{x},\mathbf{k}) \ . \tag{1}$$

The quantity

$$V_{\nu_a i} = \langle \Psi_i | \mathcal{H}_{\nu_a i} | \Psi_i \rangle_T \tag{2}$$

is the ν_a-potential in a medium composed only by the i-th matter component. In the above $\mathcal{H}_{\nu_a i}$ is the interaction Hamiltonian density and $\langle \Psi_i | ... | \Psi_i \rangle_T$ stands for the thermal average over the i-th particle component of the plasma. It should be noted by the reader that, in order to account for neutrino oscillations, in the equation (1) the integral over the neutrino momentum contains the probability $P_{\nu_a\nu_b}(\mathbf{x},\mathbf{k}) \equiv \mathbf{P}(\nu_\mathbf{a} \to \nu_\mathbf{b})(\mathbf{x},\mathbf{k})$. As we assume isotropic distributions for the neutrinos and the matter components and we disregard possible effects due to the polarization of the medium, it is easy to verify that in the one-loop approximation $V_{\nu_a i}$ does not depend on the neutrino momentum $\mathbf{k}$. In this approximation we find that the force per unit volume produced by all neutrino species on the i-th matter component is

$$\begin{aligned} \mathcal{F}_i(\mathbf{x}) &= -\nabla \mathcal{U}_i(\mathbf{x}) \\ &= \mathcal{F}_i^{osc}(\mathbf{x}) - \sum_{\mathbf{a},\mathbf{b}=\mathbf{e},\mu,\tau} \int \frac{\mathbf{d^3k}}{(\mathbf{2}\pi)^\mathbf{3}} \mathbf{P}_{\nu_\mathbf{a}\nu_\mathbf{b}}(\mathbf{x},\mathbf{k}) \nabla \left[\mathbf{f}_{\nu_\mathbf{a}}(\mathbf{x};\mathbf{k}) \mathbf{V}_{\nu_\mathbf{b}\mathbf{i}}(\mathbf{x}) \right] \end{aligned}$$

where

$$\mathcal{F}_i^{\rm osc}(\mathbf{x}) = -\sum_{a,b=e,\mu,\tau} V_{\nu_b i}(\mathbf{x}) \int \frac{d^3\mathbf{k}}{(2\pi)^3} f_{\nu_a}(\mathbf{x};\mathbf{k}) \nabla P_{\nu_a\nu_b}(\mathbf{x},\mathbf{k}). \tag{3}$$

The latter contribution $\mathcal{F}_i^{\rm osc}$ is the new term induced by the oscillations, or conversions, of a neutrino species into another – active or sterile. Clearly, that expression for $\mathcal{F}_i^{\rm osc}$ in Eq. (3) accounts for whatever kind of neutrino conversion once the proper probability is specified. We note that in the absence of neutrino oscillations our expression (3) reproduces the results of Hardy and Melrose [4, 11].

We predict several interesting effects of the ponderomotive force induced by neutrino oscillations. First of all, this force will produce a rearrangement in the density of the different matter components. Such an effect will be maximal in the case of MSW (or RSFP) neutrino resonant conversion. Indeed, in the resonance layer the neutrino survival probability undergoes a rapid variation which may give rise to strong macroscopic forces. For this reason, this is the case that we are going to discuss in more details here. As a secondary effect, the density modulation produced by the force will affect the neutrino propagation and the neutrino oscillations/conversion. We shall not study this second order effect here. However, it is worthwhile to observe that as a consequence of this effect and of its feedback on the ponderomotive force, non-linear effects appear which may lead to a sizeable energy transfer from the neutrino to the plasma [10] and to the amplification (or the fast damping) of the neutrino oscillations.

1.*Ponderomotive force due to MSW resonant conversion.* It is worth recalling the main features of the resonant neutrino conversion. We consider the system of two neutrinos ν_e and ν_x ($x = \mu, \tau, s$) (here ν_s denote an iso-singlet state), characterized by the difference of the eigenstate mass squares $\delta m^2 \equiv m_2^2 - m_1^2$ and the vacuum mixing angle θ. The coherent neutrino scattering off matter constituents can be described in terms of matter potentials as shown in Eq. (2). In the rest frame of the unpolarized matter, they read as:

$$V_{\nu_e e^\mp} = \pm \frac{G_F}{\sqrt{2} m_n}(4\sin^2\theta_w + 1)\rho Y_{e^\mp}, \tag{4}$$

$$V_{\nu_{\mu,\tau} e^\mp} = \pm \frac{G_F}{\sqrt{2} m_n}(4\sin^2\theta_w - 1)\rho Y_{e^\mp}, \tag{5}$$

$$V_{\nu_a p} = \frac{G_F}{\sqrt{2} m_n}(1 - 4\sin^2\theta_w)\rho Y_p \ , \tag{6}$$

$$V_{\nu_a n} = \frac{G_F}{\sqrt{2} m_n}\rho Y_n, \qquad V_{\nu_s i} = 0 \ , \qquad\quad i = e^\mp, p, n \tag{7}$$

here ρ is the matter density, Y_i the concentration of the i-type component and m_n the nucleon mass (for the anti-neutrinos, the above potentials change sign). The Hamiltonian $\mathbf{H}$ governing the evolution equations in matter can be written as:

$$\begin{aligned} \mathbf{H} &= \begin{pmatrix} \frac{\delta m^2}{2E}\cos 2\theta - \Delta V & \frac{\delta m^2}{4E}\sin 2\theta \\ \frac{\delta m^2}{4E}\sin 2\theta & 0 \end{pmatrix} , \\ \Delta V &= \sum_i (V_{\nu_e i} - V_{\nu_x i}) = \sqrt{2}\frac{G_F}{m_n}\rho_{eff}(r) \end{aligned} \tag{8}$$

here ΔV is the effective matter potential for the system of $\nu_e - \nu_x$ and $\rho_{\text{eff}} = \rho Y$ where the net effective concentration Y is $Y = Y_e \equiv Y_{e^-} - Y_{e^+}$ for the $\nu_e - \nu_{\mu,\tau}$ system and $Y = Y_e - \frac{1}{2}Y_n$ for the $\nu_e - \nu_s$ channel. The mixing angle θ_m and the neutrino wavelength λ_m in matter are given by:

$$\sin^2 2\theta_m = \frac{(\delta m^2 \sin 2\theta)^2}{(\delta m^2\cos 2\theta - 2E\Delta V)^2 + (\delta m^2 \sin 2\theta)^2} \ , \tag{9}$$

$$\lambda_m = \frac{\delta m^2 \lambda}{\sqrt{(\delta m^2 \cos 2\theta - 2E\Delta V)^2 + (\delta m^2 \sin 2\theta)^2}} \tag{10}$$

where $\lambda = \dfrac{4\pi E}{\delta m^2}$ is the vacuum wavelength.

As it is well known the efficiency of the conversion is determined by the resonance condition, $\dfrac{\delta m^2 \cos 2\theta}{2E} = \Delta V$ and by the adiabaticity property $\dfrac{\mathrm{d}\theta_\mathrm{m}}{\mathrm{dr}} \ll \pi/\lambda_m$ where

$$\frac{\mathrm{d}\theta_\mathrm{m}}{\mathrm{dr}} = \sin^2 2\theta_m \, \frac{E\Delta V}{\delta m^2 \sin 2\theta} h^{-1}(r) \ , \tag{11}$$

$$h^{-1}(r) \equiv \frac{\mathrm{d}\ \ln(\rho_\mathrm{eff})}{\mathrm{dr}} \ . \tag{12}$$

The ν_e survival probability at a certain distance r from the source is given by

$$P_{\nu_e}(r;E) = \frac{1}{2} + (\frac{1}{2} - P_c) \cos 2\theta_m(r_i) \cos 2\theta_m(r) \ , \tag{13}$$

where r_i is the (radial) coordinate of the neutrino source and the function P_c is the probability of jumping from one matter eigenstate to the other [13]. In typical cases, $E\Delta V \gg \delta m^2$ in the neutrino production region which implies $\cos 2\theta_m(r_i) \approx -1$. We assume the resonant conversion to be adiabatic $-P_c \simeq 0$ (this will be justified later on). Then, from Eqs. (11) and (13) we can easily obtain

$$\frac{\partial P_{\nu_e}}{\partial r} = \sin 2\theta_m \frac{\mathrm{d}\theta_\mathrm{m}}{\mathrm{dr}} = \sin^3 2\theta_m \frac{E\Delta V}{\delta m^2 \sin 2\theta} h^{-1}(r) \ . \tag{14}$$

We note that the r.h.s. of Eq.(14) attains a maximum when the resonance condition is fulfilled ($\sin^2 2\theta_m = 1$).

Now we are ready to turn back to the expression for the ponderomotive force induced by the neutrino resonant conversion given in Eq. (3). For simplicity, we assume that only ν_e's are created by some point-like source with a thermal distribution $f_{\nu_e}(r;E)$ and propagate isotropically trough an inhomogeneous medium like that e.g. present in a star. Expressing the neutrino density in term of the luminosity L_ν ($n_\nu = \dfrac{L_\nu}{4\pi r^2 \langle E \rangle}$) we find from Eq. (3)

$$\begin{aligned} \mathcal{F}_\mathbf{i}^\mathbf{osc}(\mathbf{r}) &= -(V_{\nu_e i} - V_{\nu_x i}) \frac{L_\nu}{4\pi r^2} \frac{\int dE E^2 f_{\nu_e}(E) \frac{\partial P_{\nu_e}(r,E)}{\partial r}}{\int dE E^3 f_{\nu_e}(E)} \\ &= -(V_{\nu_e i} - V_{\nu_x i}) \frac{\Delta V}{\delta m^2 \sin 2\theta} \frac{L_\nu}{4\pi r^2} h^{-1}(r) I(r) \end{aligned} \tag{15}$$

where

$$\mathrm{I}(r) \equiv \frac{\int dE E^3 \sin^3 2\theta_m(r,E) f_{\nu_e}(E)}{\int dE E^3 f_{\nu_e}(E)} \ . \tag{16}$$

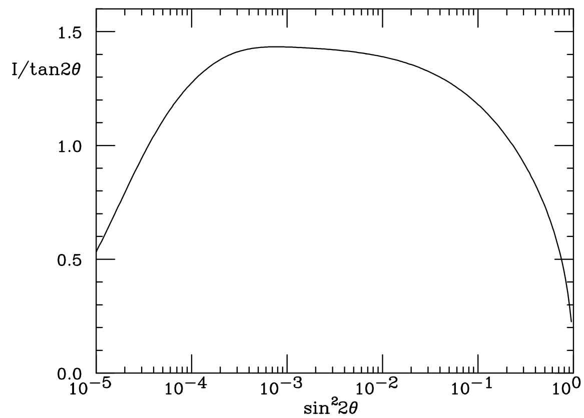

Figure 1: The ratio $I_*(\theta)/\tan 2\theta$ is plotted as a function of $\sin^2 2\theta$.

Since the function I is obtained by a convolution of $\sin^3 2\theta_m(E)$ with the neutrino distribution function, it attains the maximum value I$_*$ at the position $r_* \equiv r_{\rm res}|_{E=\langle E\rangle}$, where $\langle E\rangle$ is the neutrino mean energy. The actual value of I$_*$ depends on the width $\Delta E \propto \sin 2\theta \dfrac{\delta m^2}{\Delta V}$ of the function $\sin^3 2\theta_m(E)$, hence on the degree of adiabaticity of the neutrino conversion. Therefore the maximal force is

$$\mathcal{F}_i^{\rm osc}(r_*) = -(V_{\nu_e i} - V_{\nu_x i})\frac{L_\nu}{8\pi r^2\langle E\rangle} h^{-1}(r_*)\frac{I_*(\theta)}{\tan 2\theta} \ . \tag{17}$$

By comparing this expression with that of the conventional force driven by the gradient of the neutrino density (i.e. the second term in eq. (3), hereafter named $\mathcal{F}^{HM}$) we note a significative difference. While the latter is proportional to the net neutrino density N_ν, the former depends only on the helicity species which undergoes the resonant conversion. Such a different behavior may play a crucial role in SNs and in other astrophysical or cosmological environments. Upon making the comparison more quantitatively, we consider the forces produced on the electron component of the medium. We find

$$\left.\frac{\mathcal{F}_e^{\rm osc}}{\mathcal{F}^{HM}}\right|_{r_*} \approx \frac{r_* h_*^{-1}}{4\xi}\frac{I_*(\theta)}{\tan 2\theta} \ , \tag{18}$$

where the parameter $\xi \equiv 1 - \frac{\langle E\rangle}{\langle \bar{E}\rangle}$ accounts for the possible difference in the ν_e and $\bar{\nu}_e$ spectra (we assume that $L_{\nu_e} = L_{\bar{\nu}_e}$). In the Fig.1 we plot the function $I_*(\theta)/\tan 2\theta$. We observe that this function does not lead to any suppression in the phenomenological interesting range $10^{-4} \lesssim \sin^2 2\theta \lesssim 2 \times 10^{-1}$. The actual value of $r_* h^{-1}$ depends on the matter density profile and on the kind of neutrino transition (active-active or active-sterile) considered. Below we will show as in a SN such a quantity typically ranges from a values of few units to some powers of 10. We should keep in mind, however, that the adiabaticity requirement implies an upper limit to the allowed value of h^{-1}. This is given by

$$h^{-1} \ll h^{-1}_{\rm adiab} = \frac{\delta m^2 \sin^2 2\theta}{2\langle E\rangle \cos\theta} \ . \tag{19}$$

Above this value the scale setting the gradient of P_{ν_e} is λ_m which, in the non-adiabatic regime, is larger than h. Therefore the maximal force is achieved in the adiabatic regime.

For completeness, we also compare the force $\mathcal{F}_{\mathbf{e}}^{\rm osc}$ with the outward force due to the elastic scattering off electrons, $\mathcal{F}_e^{\rm scatt} \sim \dfrac{G_F^2}{2\pi^2}\dfrac{L_\nu}{4\pi r^2}\langle E\rangle T_e n_e$. We find

$$\left.\frac{\mathcal{F}_{\mathbf{e}}^{\rm osc}}{F_e^{\rm scatt}}\right|_{r_*} \approx 8\times 10^{-7}\left(\frac{1{\rm MeV}}{T_e}\frac{100{\rm MeV}^2}{E^2}\frac{h^{-1}}{10^{-5}{\rm cm}^{-1}}\right)\frac{I_*(\theta)}{\tan 2\theta} \tag{20}$$

where T_e is the electron temperature. Clearly $\mathcal{F}_{\mathbf{e}}^{\rm osc} \ll \mathcal{F}_{\mathbf{e}}^{\rm scatt}$. We observe, however that these forces are qualitatively quite different and cannot be always directly compared. For example, the ponderomotive force naturally excite acoustic and plasma waves which can hardly be done by the force due to the incoherent neutrino scattering. Therefore the former may induce effects which are not produced by the latter allowing its possible identification.

2.*Ponderomotive force in supernova.* As an application of our previous results, we now investigate what kind of effects the ponderomotive forces produced by neutrino resonant conversion may give rise to during a type II SN explosion. We consider the region above the neutrino sphere at a time after the core bounce. In this epoch all the neutrino (as well anti-neutrinos) species are emitted with approximately the same luminosity. However the individual neutrino energy distributions may be quite different. For this reason the force produced, say by the $\nu_e \to \nu_\mu$ MSW conversion is not canceled by the opposite force produced by $\nu_\mu \to \nu_e$ that will occur more externally. In both cases the resonance takes place in a dynamically relevant region for $\delta m^2 = 10 \div 10^4$ eV^2 if $\langle E\rangle \simeq 10$ MeV. In the earlier epoch ($t \lesssim 1$ s) with a typical profile [14] $N_e = 10^{34} r_7^{-3}$ cm^{-3} ($r_7 \equiv r/10^7$ cm), from eq.(18) we find $\left.\dfrac{\mathcal{F}_{\mathbf{e}}^{\rm osc}}{\mathcal{F}^{\mathcal{HM}}}\right|_{r_*} \sim 1$ with $\xi \sim 0.3$. At later times, as the scale h becomes smaller, this ratio can increase up to an order of magnitude.

A different picture may emerge for the $\nu_e - \nu_s$, or the $\bar\nu_e - \bar\nu_s$, channels. We recall that in this case $\rho_{\rm eff} = \rho(3Y_e - 1)$ and the resonance condition can be satisfied for arbitrarily small values of δm^2. Hence, for small values of δm^2 the resonance take place nearby the point where the potential approaches zero. As a consequence $h^{-1}(r_*)$ could be quite large. At the same time, however, the adiabaticity requirement (19) set an upper limit to h^{-1} which is proportional to δm^2. Adopting standard SN density profile [15] and assuming $\sin^2 2\theta \lesssim 10^{-2}$, we found that the maximal force is obtained for $\delta m^2 \sim 1$ eV^2, corresponding to $\Delta V \simeq 10^{-7}$ eV and $h^{-1} \sim 10^{-5}$ cm^{-1} ($r_* \simeq 150$ km). Therefore the ratio (18) can be as large as 10^2.

A comment at this point is in order. Because the electron neutrino potential changes sign at the radius where $Y_e = 1/3$, two kinds of resonance should take place close to this position. If we assume $0 < \delta m^2 \lesssim 10^2$ eV^2, and since Y_e is a growing function of the radius above the neutrino-sphere, the transition $\bar\nu_e - \bar\nu_s$ will first occur in the region where $Y_e < 1/3$. It is easy to verify that in this case the force pulls electrons inwards. Soon afterwards the transition $\nu_e - \nu_s$ take place in the region with $Y_e > 1/3$ and thereby the force goes in the opposite direction. It happens [15] that the two types of conversion

take place very close to each other. However the corresponding forces should not cancel as the average energy of ν_e and $\bar{\nu}_e$ differs of some 30-50%. In the case $\delta m^2 > 10^2$ eV^2 only the conversion $\bar{\nu}_e - \bar{\nu}_s$ can happen as far as $Y_e < 1/3$ [15].

It is interesting to estimate the velocity which background matter close to the resonance position acquires under the action of $\mathcal{F}_{\mathbf{e}}^{\mathrm{osc}}$ in a time interval of the order of the dynamical time $\simeq 0.1$ s. A straightforward computations shows that velocities as large as $\sim 10^2$ km/s can be reached. Other interesting effects arise if these ponderomotive forces induce plasma instabilities [10] which may give rise to a significative energy transfer from the neutrino to the plasma.

In conclusion, we think that the phenomenological perspectives open by the study of the ponderomotive force produced by neutrino oscillations are rich and deserve further investigation.

Acknowledgements We thank M. Pietroni for helpful discussions.

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